Multi-scale Dictionaries Based Fingerprint Orientation Field Estimation

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Abstract

Orientation field estimation is significantly important for fingerprint recognition. Dictionary based algorithm and its variant, localized dictionaries based algorithm have shown promising performance. In this paper, we extend the original dictionary based algorithm to a multi-scale version. The motivation is that small scale dictionary is more accurate while large scale dictionary is more robust against image noise. Hence information from orientation fields of different scales can be integrated to obtain better results. A multi-layer MRF model is used to formulate and solve the proposed problem. Experimental results on challenging latent fingerprint database demonstrate the advantages of the proposed algorithm.

1. Introduction

Due to the uniqueness and persistence of fingerprint, fingerprint recognition has been applied to many fields, such as forensics, customs security and personal device login. A typical fingerprint recognition systems contains the following steps: 1) the fingerprint image is enhanced to remove noise, connect broken ridges and separate joined ridges; 2) robust features, such as minutiae, are then extracted from the enhanced image; 3) a matching score is computed by comparing minutiae. Successful image enhancement algorithms [13, 17, 9, 10, 18] require reliable orientation field estimation. Conventional local ridge orientation estimation methods, for example, Short Time Fourier Transform (STFT) [5], calculate ridge orientation based on a small area of image patch, which is sensitive to background noise. Feng *et al*. propose a fingerprint orientation field estimation algorithm based on prior knowledge of fingerprint structure [7], also known as dictionary based orientation filed estimation or GlobalDict, which has shown promising performance for latent fingerprints. Yang *et al*. extend this idea and take stronger prior knowledge, specifically, pose of fingerprint, into account [22] (hereinafter referred to as localized dictionaries based method or LocalDict for short). The introduction of position-dependent dictionaries avoids the impossible occurrence of orientation patches and thus obtains better performance. However, both GlobalDict and LocalDict encounter inaccurate estimation results around singularities.

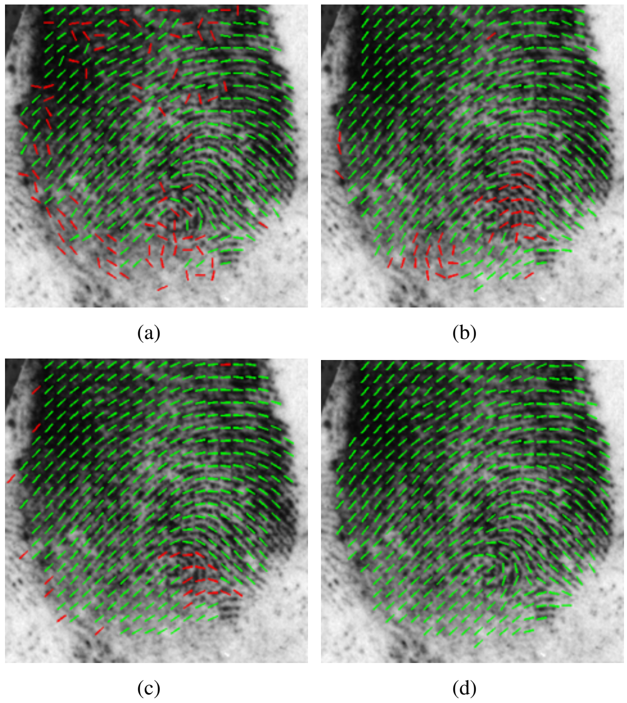

Figure 1. Orientation fields of a latent fingerprint in NIST SD27 obtained by (a) STFT [5], (b) GlobalDict [7], (c) LocalDict [22] and (d) manually method (ground truth), respectively. Orientations different from ground truth by more than 20 degrees are denoted by red lines. The orientation field in (a) is quite noisy. Estimation results by dictionary based algorithms are smooth but inaccurate around singular points.

Figure 1 shows the orientation fields of a latent print estimated by four different methods, namely STFT (Figure 1(a)), GlobalDict (Figure 1(b)), LocalDict (Figure 1(c)) and manually marking (Figure 1(d)). Orientation field ob-

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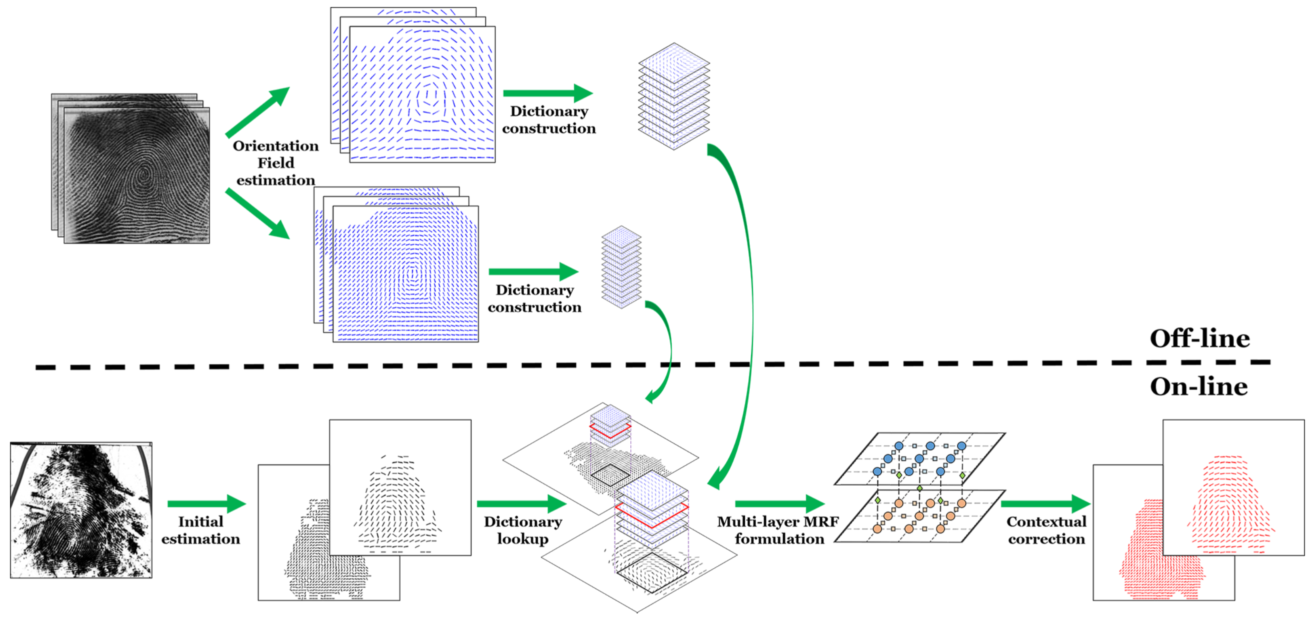


Figure 2. The flowchart of the proposed algorithm, including an off-line dictionary construction module and an on-line estimation module.

tained by STFT is very sensitive to noise. The results estimated by GlobalDict and LocalDict are smooth and robust against image noise, but inaccurate near the singular points.

All the algorithms mentioned above consider a single scale orientation field. Here the scale of an orientation field refers to the image block size for each orientation element. Small scale orientation field corresponds to small image block size. In this paper, we propose a multi-scale dictionaries based fingerprint orientation field estimation algorithm.

The scale of noise in latent fingerprints varies a lot. Our analysis results on a public latent fingerprint database, NIST SD27, show that the scale of noise of good-quality fingerprints, bad-quality fingerprints and ugly-quality fingerprints are 7.46, 11.62, and 19.55 blocks (16×16 pixels), respectively, which are computed as the average size of the connected domain in the noise areas. It means that prints of lower quality have larger scale of image noise and to deal with fingerprints of different quality, dictionaries of different scales should be considered. In other words, by introducing multi-scale model for orientation field estimation, robustness of the results against image noise could be increased.

Another motivation of this paper is to increase the orientation field estimation accuracy around singular points. An obvious fact in orientation field estimation is that, small scale orientation field is more accurate than large scale orientation field, especially near the singular points. This suggests us that multi-scale orientation fields can be integrated to achieve more robust and accurate results.

Previously, multi-scale image dictionaries has been proposed by Liu *et al*. for fingerprint enhancement [12]. In this paper we use orientation dictionaries which can cover larger fingerprint area than ridge dictionaries and thus should be more robust to noise.

The outline of the proposed algorithm is shown in Figure 2, including an off-line dictionary construction module and an on-line estimation module. In the off-line part, multi-scale orientation patch dictionaries are learnt independently. In the on-line stage, fingerprint orientation field is estimated by three main steps, namely, initial orientation field estimation, multi-scale dictionaries lookup, multi-layer MRF formulation and contextual correction.

The rest of the paper is organized as follows. Section 2 will be a brief review of published orientation field estimation algorithms. In Section 3, details of the proposed algorithm will be described. Experimental results will be reported in Section 4 and finally in Section 5, we will summarize the paper.

2. Related Work

In this section, we briefly review the representative algorithms for fingerprint orientation field estimation.

Local estimation approaches compute a local ridge orientation using only the neighborhood pixels around. Gradient, silt-based and local Fourier analysis are three most representative methods. Gradient based approaches compute local orientation by summarizing gradients of local neighborhood [2]. Silt-based methods choose the direction with smallest intensity variation as the local orientation [14]. Differently, local Fourier analysis applies Fourier transform on a local image patch and analyzes in the frequency domain to find out the local orientation. All the three kinds of methods mentioned above consider small local area, typically 32×32 or 64×64 pixels for computing a local orientation and thus are sensitive to image noise.

To deal with this problem, a number of regularization techniques have been proposed. The most simple approach is to apply low-pass filtering on the initial orientation field [2], considering the fact that orientation field is smooth in most area. Another regularization technique is MRF-based. The orientation field is modeled as Markov Random Field [10, 6, 11] and optimized using graph cut [4] or loopy belief propagation [3] to minimize the energy function.

Different from the local estimation approaches, global parametric models attempt to solve this problem using general functions, such as polynomials [8, 15] and Fourier series [21]. The expressiveness of the general models depend on their complexities, for example, the order of the polynomials. However, increasing expressiveness without necessary constraints on parameters usually results in overfitting. Additionally, some models require singular points as input, while detection of singular points relies on accurate orientation field.

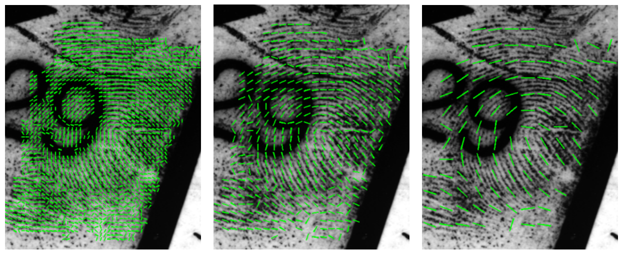
Recent studies introduce prior knowledge of fingerprint orientation fields learnt from many fingerprint orientation fields as new regularization method for orientation field. Dictionary based algorithm proposed by Feng *et al*. [7] and localized dictionaries based method proposed by Yang *et al*. [22] are two representative approaches. Feng *et al*. constructed an orientation patch dictionary from a set of manually marked orientation fields, so that noisy orientations can be replaced by the closest orientation patches found in the dictionary. Yang *et al*. extended this idea and constructed localized dictionaries by introducing fingerprint pose and spatial constraint. That is, constructing a set of independent dictionaries of orientation patches for each location, so that orientation patches which are not likely to be in that location will not be looked up.

Dictionary based and localized dictionaries based algorithms both show promising performance in correcting and smoothing fingerprint orientation fields. However, as mentioned previously, both approaches usually result in inaccuracy around singularities. This motivates us to consider multi-scale prior knowledge of fingerprint orientation fields. In the following section, we will present our algorithm in detail.

3. Proposed Algorithm

3.1. Constructing Multi-scale Dictionaries

Similar to [7], we construct dictionaries of orientation patches from a set of high-quality fingerprints from NIST SD4, a public rolled SD database. Three different scales of



(a) (b) (c)

Figure 3. Initial orientation field of three different scales with block size of 8×8, 16×16 and 32×32 respectively.

orientation fields are defined on image blocks of size 8×8 pixels, 16×16 pixels and 32×32 pixels, respectively. For each training fingerprint image, three different scales of orientation fields are manually marked as ground truth. Orientation patches are extracted by sliding a fixed size window ($N_p \times N_p$ blocks) across each reference orientation field of each scale. The orientation patch size parameter N_p has significant effect on the expressiveness of dictionary. Larger N_p usually means stronger regularization. However, for dictionary with larger patch size, to achieve comparative expressiveness as small patch size, much more training patches are desired. Note that as the orientation patches of different scales have different scale of view, their distribution are different. Thus we construct separate dictionaries for each scale using the corresponding orientation patches of same scale. Refer to [7] for more details of the dictionary constructing process.

A weakness in the above dictionary constructing process is that the training orientation patches contain few patches around singularities, which could lead to wrong query results. To increase the representativeness of the dictionaries, we include more singularity orientation patches for training. As manually marking orientation field is far more difficult and time-consuming than marking the locations of loop points and delta points, we use the latter information to synthesize orientation fields and extract orientation patches. This step is realized using the orientation model proposed by Sherlock and Monro [16] or a subsequent variant model proposed by Vizcaya and Gerhardt [20]. That is, given $\mathbf{ls}_i, i = 1 \cdots n_c$ and $\mathbf{ds}_i, i = 1 \cdots n_d$ as the coordinates of the loops and deltas, the orientation θ at each point $\mathbf{z} = [x, y]$ is calculated as

$$\theta = \frac{1}{2}\left[\sum_{i=1}^{n_d} arg(\mathbf{z} - \mathbf{ds}_i) - \sum_{i=1}^{n_c} arg(\mathbf{z} - \mathbf{ls}_i)\right], \quad (1)$$

where the function $arg(\mathbf{c})$ returns the phase angle of the complex number $\mathbf{c}$.

Orientation patches can be extracted from the synthetic orientation field similarly with a fixed size sliding window.

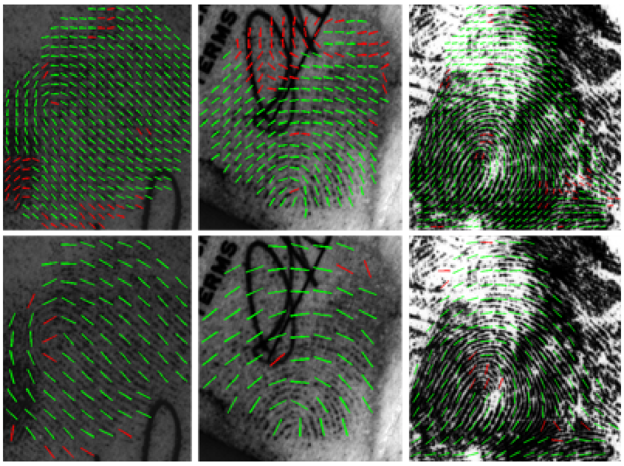


Figure 4. Fingerprint orientation fields estimated by dictionary lookup only. The first row are orientation fields with block size of 16×16 pixels and the second row are orientation fields with block size of 32×32 pixels. Orientations different from ground truth by more than 20 degrees are denoted by red lines. Orientation fields of different scales are complementary in noise areas and near singular points.

The synthetic orientation patches together with the manually marked orientation patches are then used for constructing dictionaries.

3.2. Initial Orientation Field Estimation

The initial orientation field could be obtained by any local estimation algorithms, such as STFT, gradient-based and silt-based. In the proposed framework, different scales of initial orientation fields need to be estimated in the initial step. A local orientation in a larger scale orientation field is supposed to have a larger view of image blocks than a local orientation in a small scale orientation filed. Three different scales of initial orientation fields of a latent fingerprint estimated by STFT is shown in Figure 3. Figure 3(a) is the orientation field of smaller scale, which is sensitive to noise but accurate near the loop point. Figure 3(c) is the orientation field of larger scale and is much more smooth.

One way to estimate different scales of orientation fields is separately applying short time Fourier transform to local image blocks of different sizes. In this paper, we adopt another scheme. The orientation field of the smallest scale (with block size of 8×8 for example) is firstly estimated using STFT independently. As a block in the orientation field of larger scale (with block size of 16×16 for example) corresponds to four blocks in the smaller scale one, the Fourier spectrum of the small scale orientation field can be used to analyze the larger scale orientation field. Actually, the Fourier spectrum of a print image can be viewed as a distribution of surface wave [5]. Thus the Fourier spectrum of the four small blocks are superposed to obtain Fourier spectrum of the large block. The local orientation in the larger scale orientation field then can be estimated by searching the maximum response in the new obtained Fourier spectrum.

Table 1. Average time (in seconds) for estimating initial orientation field using the separate scheme and the combined scheme.

Block size = 8	Block size = 16	Block size = 32	Separate scheme	Combined scheme
21.45	8.63	5.48	35.56	22.70

One advantage of the above initial orientation field estimation scheme is that it avoids a lot of Fourier transform computation and thus is more efficient. The average time for estimating initial orientation field using the two methods mentioned above are listed in Table 1. The first three columns are the average time for estimating the initial orientation field of three different scales. Small scale orientation field requires much more computation and thus is much more time-consuming than large scale orientation field. The fourth column is the total time of the first three columns. Our proposed method (referred as combined scheme) for estimating initial orientation field mentioned above costs 22.70 seconds in average, 36% faster than the separate approach.

3.3. Dictionary Lookup

Dictionary lookup step searches orientation patches in the pre-trained dictionaries which are most closest to the initial orientation patches. Orientation fields of different scales look up the corresponding dictionary of the same scale, independently. Contextual information between different scales is left to the later stage. For each orientation patch in the initial orientation field of an exact scale, at most N_c (N_c is set to 6 in this paper) orientation patches in the corresponding dictionary with highest scores are selected as candidates, with consideration of diversity examination mentioned in [7].

Figure 4 shows three examples of orientation fields estimated by dictionary lookup only. Orientation fields of different scales are complementary in different areas. Small scale orientation fields perform better near the singular points. Large scale orientation fields are more smooth and robust against image noise. This complementarity explains that integrating orientation fields of different scales is necessary and should be effective. The following subsection will introduce the details for integrating information from different scale orientation fields.

3.4. Contextual Orientation Field Correction

Contextual information has been adopted for many tasks, such as natural language processing and image segmentation, and has been proved to be effective for fingerprint orientation filed correction. After dictionary lookup, at most

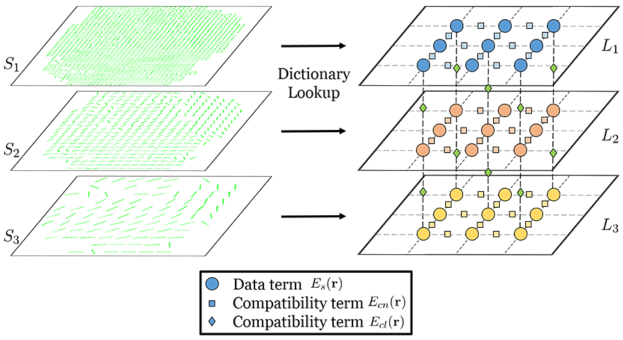


Figure 5. Multi-layer MRF model for multi-scale orientation fields correction. The left side is initial orientation fields. After dictionary lookup stage, the orientation field estimation problem is formulated as an multi-layer MRF model shown in the right side. The data term and the two compatibility terms are represented in different shapes.

N_c orientation patches are found as candidates for each initial patch. Suppose that we have N_l different scales of orientation fields. Our target is to determine a single candidate for each patch in each scale, taking both compatibility of neighborhood and across different scales into account.

As Figure 5 illustrates, the problem can be formulated as a multi-layer MRF model. During the dictionary lookup stage, we slide the window with fixed pixels for orientation fields of different scales, so that different layers have the same number of nodes.

The multi-layer MRF model then can be optimized by minimizing an energy function $E(\mathbf{r})$ defined as

$$E(\mathbf{r}) = E_s(\mathbf{r}) + \omega_{cn} E_{cn}(\mathbf{r}) + \omega_{cl} E_{cl}(\mathbf{r}), \qquad (2)$$

where $E_s(\mathbf{r})$ denotes the similarity between the candidate orientation patch and the corresponding initial patch, $E_{cn}(\mathbf{r})$ denotes the compatibility between neighboring candidate patches and $E_{cl}(\mathbf{r})$ refers to the compatibility across different layers. ω_{cn} and ω_{cl} are the weights of the compatibility terms.

The three terms in (2) are illustrated in Figure 5 using different shapes. The definition of the first two terms $E_s(\mathbf{r})$ and $E_{cn}(\mathbf{r})$ in the above energy function are the same as that in [7].

The compatibility across different layers is measured by the similarity of orientation patches of different scales. Figure 6(a) and Figure 6(b) show two orientation patches obtained from dictionaries of different scales. Though the patch size of the two orientation patches are of the same (4×4 as in Figure 6), orientations in different scales have different scales of view. To measure their similarity, we firstly upsampling the orientation patch of large scale (Figure 6(b)) using nearest neighbor interpolation to obtain a new orientation patch that has same scale with the small

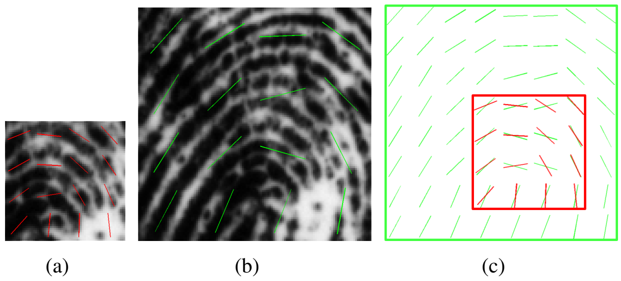


Figure 6. Compatibility between orientation patches of different scales in the same location. (a) An image patch and the orientation patch obtained by dictionary lookup with block size of 16×16. (b) An image patch and the orientation patch obtained by dictionary lookup with block size of 32×32. (a) and (b) are in the same location of a same print image. (c) Illustration of computation of the compatibility between the two orientation patches.

scale one. The compatibility is then computed by the similarity of the overlapping blocks, as Figure 6(c) illustrates. Let $\{\theta_{1n}\}_{n=1}^{N_o}$ and $\{\theta_{2n}\}_{n=1}^{N_o}$ be the set of orientations in the overlapping blocks. The similarity is defined as

$$C\left(\mathbf{\Phi}_{i,p,l_1}, \mathbf{\Phi}_{i,q,l_2}\right) = \frac{1}{N_o} \sum_{n=1}^{N_o} |cos(\theta_{1n} - \theta_{2n})|, \qquad (3)$$

where $\mathbf{\Phi}_{i,p,l}$ refers to the p^{th} candidate orientation patch at location i of the l^{th} layer. The compatibility across two layers is defined as

$$E_{cl}(\mathbf{r}) = \sum_i \left(1 - C\left(\mathbf{\Phi}_{i,p,l_1}, \mathbf{\Phi}_{i,q,l_2}\right)\right). \qquad (4)$$

For model with three or more layers, $E_{cl}(\mathbf{r})$ could be computed similarly.

The energy function in (2) is solved by loopy belief propagation [3], a well-known algorithm for inference on trees and MRF models.

4. Experimental results

In this section, fingerprint orientation field estimation performance will be reported and compared with several representative algorithms.

4.1. Databases

The proposed multi-scale dictionaries are constructed with 200 good quality rolled fingerprints from NIST SD4. In addition, 2000 manually marked singularities are used to generate synthetic orientation patches around singularities. We evaluate the proposed method on the public challenging latent fingerprint database, NIST SD27.

The NIST SD27 database contains 258 latent fingerprints with very poor quality, including three subsets of different

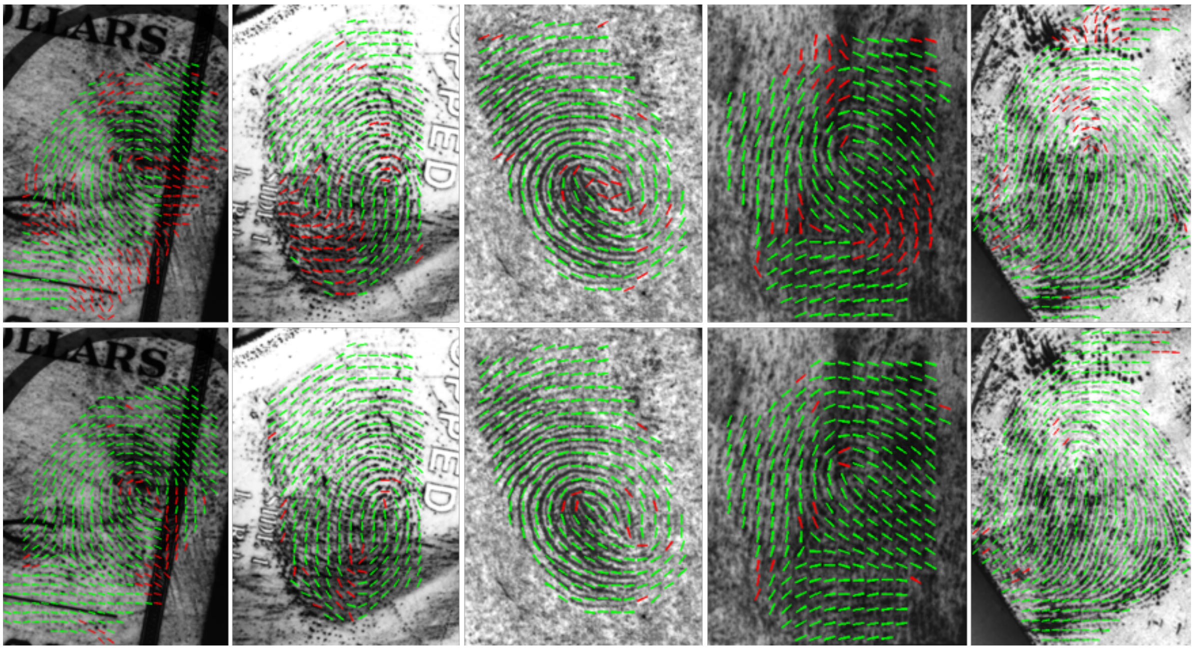


Figure 7. Comparison of fingerprint orientation field estimated by GlobalDict [7] and the proposed algorithm. The first row is obtained by GlobalDict and the second row is obtained by the proposed algorithm. Orientation fields are represented by the short lines. The red lines denote orientations different from ground truth by more than 20 degrees while the green lines refer to those different from ground truth not more than 20 degrees.

quality level, namely "good", "bad" and "ugly". The numbers of latents in the three subsets are 88, 85 and 85 respectively.

For fingerprint matching evaluation, 27000 rolled fingerprints in NIST SD14 are used as background database to make it more challenging.

4.2. Performance Evaluation

An important and direct measure of orientation estimation algorithm is the estimation accuracy compared with manually marked orientation field. Specifically, this is computed by the average Root Mean Square Deviation (RMSD). In this paper, the manually marked orientation field suggested in [19] is used as ground truth. The proposed algorithm is compared with three different algorithms (GlobalDict [7], FOMFE [21] and STFT [5]). Results on all the 258 prints in NIST SD27 and on the subsets of different quality levels are listed in Table 2. The proposed algorithm outperforms the other three algorithms on the latent database.

Further more, in order to better measure the estimation accuracy in singularities areas, the RMSD measure is calculated around the location of manually marked singular points. Specifically, for each singular point (loop or delta), we extract the 5×5 orientation patch centering at the singularity location and compute the average RMSD against the corresponding orientation patch from manually marked orientation field. The average estimation errors are shown in Table 3. The proposed algorithm also outperforms GlobalDict.

Table 2. Average estimation error (in degrees) of the proposed algorithm and three published orientation field estimation algorithms on the NIST SD27 database.

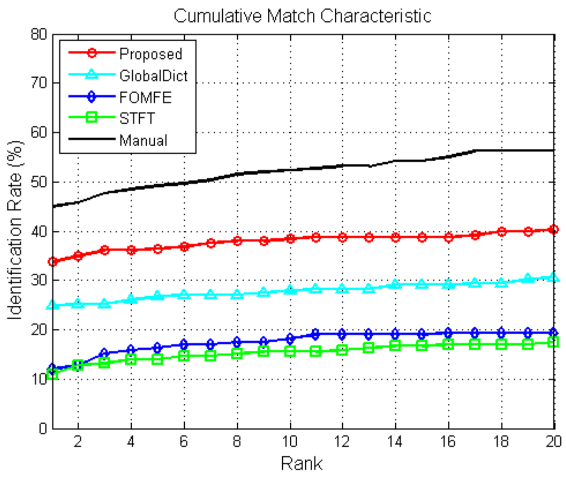
Algorithm	**All**	**Good**	**Bad**	**Ugly**
Proposed	**16.09**	**12.35**	**16.94**	**19.11**
GlobalDict [7]	18.44	14.40	19.18	21.88
FOMFE [21]	28.12	22.83	29.09	32.63
STFT [5]	32.51	27.27	34.10	36.36

Table 3. Average estimation error (in degrees) of the proposed algorithm and GlobalDict on the NIST SD27 database around singularities.

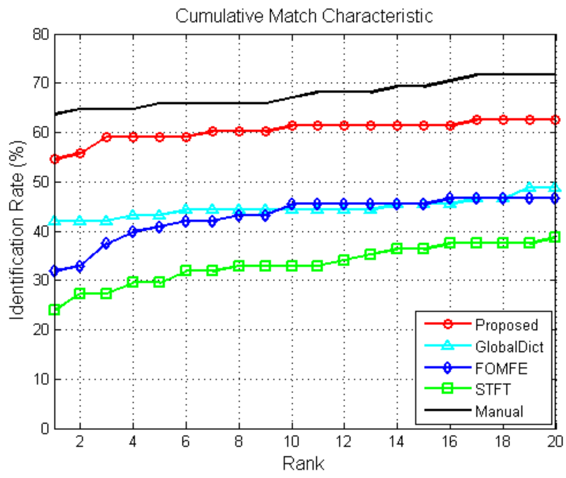
Algorithm	**Loop**	**Delta**	**All**
Proposed	17.63	14.89	16.63
GlobalDict [7]	18.68	17.88	18.39

Several examples are given in Figure 7 to compare orientation fields obtained by GlobalDict (the first row) and the proposed algorithm (the second row). Orientations that differ from manually marked orientation field are represented using red lines. As shown in Figure 7, the proposed algorithm is much more accurate than GlobalDict, especially around the singularities.

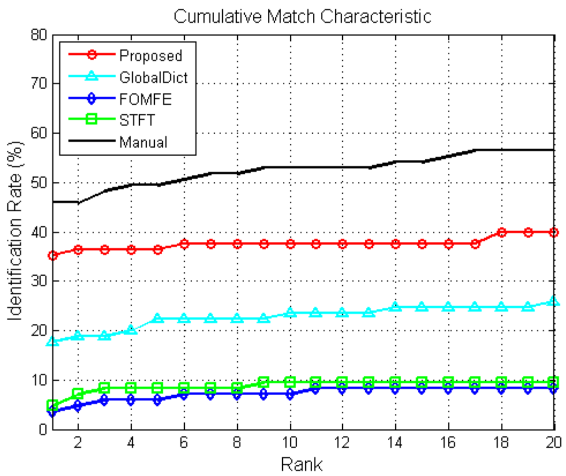
The final goal for estimating orientation field is to improve matching accuracy. Thus we conduct matching exper-



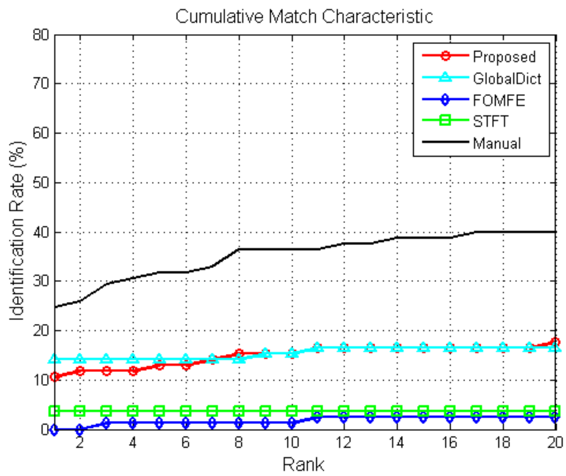
(a) All latents



(b) Good quality latents



(c) Bad quality latents



(d) Ugly quality latents

Figure 8. CMC curves of five different orientation field estimation algorithms on the NIST SD27 latent database: (a) all (258 latents), (b) good quality (88 latents), (c) bad quality (85 latents), (d) ugly quality (85 latents).

iment on the NIST SD27 database. To evaluate the matching accuracy, the latent images are enhanced using a Gabor filter with fixed frequency of $1/9$ cycles per pixel. VeriFinger 6.2 [1] is used to extract features, such as skeletons and minutiae, from enhanced fingerprints (both latent and rolled fingerprins). Note that the background database (template fingerprints in NIST SD27 and NIST SD14) could be processed off-line, so that we can use any state-of-art algorithm. In this paper, LocalDict is adopted for processing the rolled fingerprints to make the results comparable with that in [22]. Finally, the same SDK is used to compute matching scores between latent and rolled prints. We evaluate the matching performance using the Cumulative Match Characteristic (CMC) curve. Figure 8 shows the CMC curves on the NIST SD27 database and the three subsets of different quality level. As we can see from the curves, there is a clear gap between the proposed algorithm and other published algorithms (GlobalDict, FOMFE and STFT) on the whole database and the subsets of good quality and bad quality. On the ugly-quality subset, we achieve comparable performance with GlobalDict.

In general, the proposed algorithm outperforms published representative algorithms in both orientation field accuracy and fingerprint matching accuracy on public latent fingerprint database.

5. Summary and Future Work

Orientation field estimation is critical for fingerprint recognition, especially for latent fingerprint recognition. Although automatic fingerprint recognition technology has been studied for more than 40 years, matching performance for latent fingerprints is still far from satisfactory, most of time due to the severe image noise. The study in [7] opens up a new direction for this problem by introducing orientation patch dictionary as prior knowledge. Later dictionary based studies, for example, LocalDict, exploit additional prior knowledge to promote the performance. So does our study.

In this paper, we integrate information from different scale orientation fields to improve orientation estimation accuracy. Small scale orientation field provides accurate information in the high-quality areas and around singularities, while large scale orientation field is more robust against image noise. Initial orientation fields of different scales are corrected by looking up different orientation dictionaries of corresponding scales. The task is then formulated as a multi-layer MRF inference problem. Experimental results on latent fingerprint databases demonstrate the effectiveness of the proposed algorithm comparing with published representative methods.

However, the orientation estimation error of the poroposed algorithm is larger than LocalDict[22]. Combing multi-scale dictionaries with localized dictionaries is our ongoing work.

6. Acknowledgements

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