Orientation Field Estimation for Latent Fingerprints by Exhaustive Search of Large Database

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Abstract

Orientation field estimation is one of the most important steps in latent fingerprint recognition systems. However, due to very poor image quality, the performances of the state-of-the-art algorithms of orientation field estimation are still far from satisfactory. Based on the assumption that a sufficiently large fingerprint database should contain fingerprints whose orientation fields are quite similar to a latent fingerprint, we propose an orientation field estimation algorithm based on exhaustive search of large database for nearest neighbor. We set up a large database of 10,000 fingerprints of good quality, whose orientation fields and poses are estimated offline using traditional methods. Given a latent fingerprint as input, the most similar orientation field in the database is found and is combined with the original latent image to obtain the final orientation field. As a by product, we also obtain the pose of the latent fingerprint, which can be useful for fingerprint registration. The experimental results on NIST SD27 latent database show our method performs better than the state-of-the-art algorithms, in both orientation field estimation accuracy and identification performance, especially on those fingerprints of very poor quality.

1. Introduction

Latent fingerprints are now the most important and widely used evidence in criminal investigation [1]. Due to poor quality (confounding background, blurred ridge patterns, noises and so on), features (minutiae, cores and deltas) in latents are routinely manually marked by fingerprint examiners, and then they can be further handled by AFIS (automated fingerprint identification systems). The manual marking process takes too much time and energy, and cannot fulfill the need for increasing number of latent matching transactions in recent years.

In recent years, automatic latent fingerprint feature extraction has received increasing attention [2]. Due to the importance of orientation field in many fingerprint recognition tasks, researchers have proposed a number of orientation field estimation algorithms, such as dictionary-based [3, 4] and deep neural network based [5, 6, 7, 8]. However, there is still large room for improvement. For example, the lowest error, in terms of average root mean square deviation (RMSD), on NIST SD27 latent fingerprint database is 13.51 degrees (by CNN-based method [5]). Figure 1 shows the estimation result of another state-of-the-art method, LocalDict [4]. There are still some undesired errors by the method.

In this paper, we propose a new method for orientation field estimation to improve the performance. By analyzing the limitation of the previous dictionary-based methods, we find that they do not apply a constraint on the whole orientation field, and thus may produce invalid orientation fields. This limitation may be overcome by applying stronger constraint on valid orientation fields. Furthermore, there is a consensus that orientation field is not so unique as minutiae pattern. In other words, a sufficiently large fingerprint database should contain some fingerprints whose orientation fields are quite similar to a latent fingerprint. Based on the analysis above, we propose an orientation field estimation algorithm based on exhaustive search of large database. The proposed method can be seen as a member of the family of dictionary based orientation field regularization [3, 4]. The difference from these previous methods is that instead of splitting the orientation field into small patches, here the whole orientation field is used for looking up the dictionary, to make sure that the whole orientation field is valid. The proposed method can also estimate the pose of a latent fin-

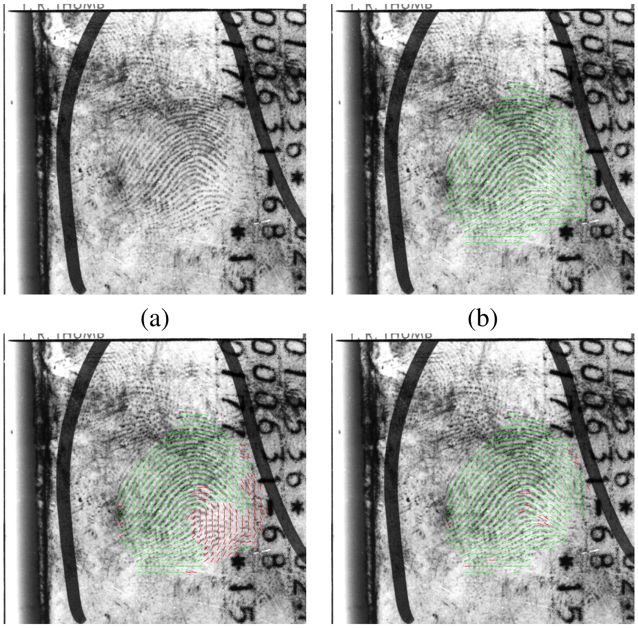

(a) (b)
(c) (d)

Figure 1. Example of orientation field estimation by different methods: (a) the original latent fingerprint image; (b) manual mark; (c) localized dictionaries based method in [4]; (d) proposed method. In (c) and (d), the green lines are correctly estimated orientation elements (difference from manual mark is less than 15 degrees), while the red lines are the estimated orientation elements with an error over 15 degrees. There are about 15% wrong elements estimated by LocalDict method and only 6% by the proposed method in this example. The RMSD of the orientation field estimated by the two methods are 19.5 and 7.2 degrees, respectively.

gerprint by the exhaustive search.

Experimental results on the NIST SD27 latent database show that the orientation estimation performance of the proposed method performs better than state-of-the-art algorithms in both orientation field estimation and latent identification accuracy, especially on those latent fingerprints of the very poor quality. The algorithm achieved an improvement of about 0.5 degrees in orientation field estimation accuracy and about 1% at rank-1 in identification rate on NIST SD27 latent database, while on the subset of ugly quality, the improvement is about 1.5 degrees and about 6%, respectively. Also, the pose estimation performance surpasses the state-of-the-art algorithm in orientation and share the same level in position.

The contributions of this paper are summarized as follows:

(1) Proposing a new method for orientation field and pose estimation using the whole fingerprint for searching instead of local patches, which can effective overcome the noises and lack of large fingerprint area.

(2) Proposing a cascade boosting method to significantly improve the search efficiency with very low loss on accuracy.

2. Related works

2.1. Orientation field estimation

Conventional methods estimate the ridge orientation by analyzing pixel values in its neighborhood (such as 16×16 or 32×32 pixels). For example, gradient-based method [9], local Fourier analysis method [10] and line-sensor method [11]. Such methods only take local image block into consider, therefore are sensitive to image quality. Therefore, they perform badly on latent fingerprints and usually get very noisy orientation fields.

Noticing the smoothness of orientation field, some researchers treat orientation field estimation as a problem of surface fitting and propose different models to describe orientation field. E.g, a fingerprint orientation model based on 2D Fourier expansion (FOMFE) by Wang et al. [12, 13], a polynomial model by Gu et al. [14], the global models based on quadratic differentials by Huckemann et al. [15]. These methods estimate parameters of models instead of orientation field itself and can partly overcome noises. But they may generate invalid orientation fields without a good constraint. Also, some of these methods need exact information of cores and deltas in fingerprints, which are very difficult to extract from latents.

In order to deal with strong noise in latents, prior knowledge of fingerprints is utilized to regularize noisy orientation fields by Feng et al. [3]. In the offline stage, they build a dictionary consisting of a set of real fingerprint orientation patches. In the online stage, noisy orientation patches are compared to those in the dictionary and are replaced by the closest one. However, the method does not take spatial distribution of orientation patches into consideration. To better model the statistics of fingerprint orientation fields, Yang et al. propose a localized dictionaries based method [4]. The method analyzes the possibility of different orientation patches at different places, and then build a set of localized dictionaries. Other researchers also use ridge pattern dictionaries for orientation field estimation [16, 17]. These dictionary based methods perform much better than conventional ones in latents, but a limitation of these methods is that different patches in one fingerprint may be inconsistent with each other. As a result, the final orientation field can may be invalid fingerprint orientation field (Figure 1(c)).

In the past few years, Convolutional Neural Networks (CNNs) have shown outstanding performance on different challenging tasks in computer vision. CNNs are considered to have strong ability to learn features directly from the original image. More and more researchers have applied deep learning for biometrics [18]. Cao and Jain pro-

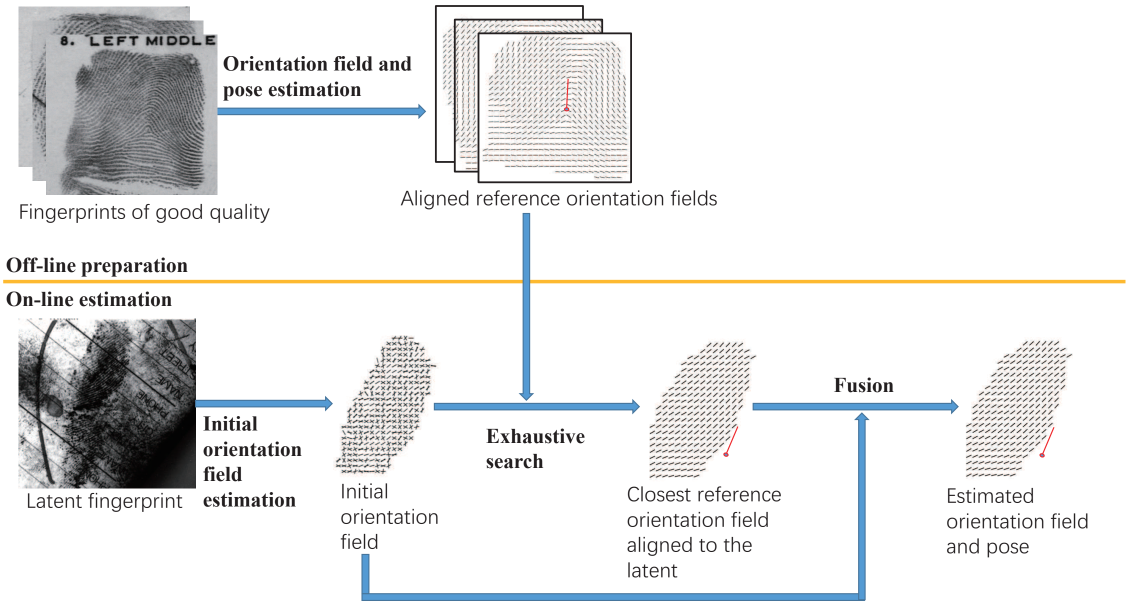


Figure 2. Flowchart of the proposed algorithm. Given a latent fingerprint as input, the algorithm outputs orientation field and pose.

pose a CNN-based method to estimate orientation fields [5]. Similar to dictionary-based methods, the CNN-based method set a database consisting of real fingerprint orientation field patches, and then train a network to classify a fingerprint image patch into corresponding orientation field patch. Tang et al. propose a unified deep network for fingerprint minutiae extraction [6], with a branch net for orientation field estimation. Also, there are more algorithms for orientation field estimation based on different methods, such as regression convnets [7] and deep expectation [8]. But their performances on the latent fingerprint database NIST SD27 are not published yet.

2.2. Pose estimation

Exact fingerprint pose estimation is valuable for several steps in fingerprint recognition systems. For example, pose constrains can effectively improve matching speed and reduce false matches in fingerprint indexing [19, 20]. Traditional fingerprint pose estimation based on special points in fingerprints, such as singular points [21] and focal points [22], is not robust for very noisy and incomplete fingerprints. Recently, learning based fingerprint pose estimation methods, which use the whole fingerprint region for estimation, achieve good performances [19, 23]. But these methods are designed for rolled fingerprints, and are sensitive to strong noises and cannot handle partial fingerprints, which are common in latent fingerprints. To our best knowledge, the Hough voting based pose estimation algorithm in [4] is the only one specially designed for latent fingerprints. But our experiments show its estimated direction is not very accurate for very incomplete latent fingerprints.

3. Proposed method

Figure 2 shows the flowchart of our method. The steps are expounded as follows.

3.1. Database preparation

The reference database D_{ref} consists of the orientation fields of fingerprints from NIST SD4 and NIST SD14. NIST SD4 is a database of 2,000 pairs of rolled fingerprints, and it has equally distributed fingerprint types (arch, tented arch, left loop, right loop and whorl). While NIST SD14 is a database of 27,000 pairs of rolled fingerprints, and the distribution of fingerprint types is similar to that in real world (fewer arch and tented arch, more loop and whorl).

We select 400 fingerprints from NIST SD4 database, and manually mark their orientation fields. To enlarge the training database, we randomly select 9,600 fingerprints in NIST SD14, and get their orientation fields by the commercial software VeriFinger SDK 6.2. By using an automatic approach, the database can be easily expanded or exchanged for different tasks. Also, we estimate the poses of these fingerprints by faster R-CNN method [23], including the finger center and orientation.

3.2. Initial orientation field estimation

Given a latent query fingerprint, the first step is to estimate the initial orientation field. In the proposed method, the initial orientation field of the latent fingerprint is esti-

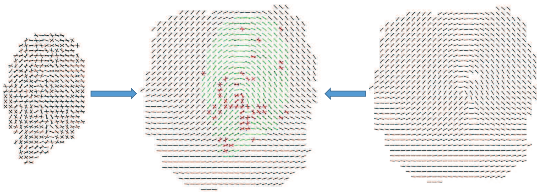


Figure 3. Example of similarity computation. An initial orientation field (red) and its closest reference orientation field (black), the places where either of the two strongest initial elements differs less than 15 degrees from the reference element are marked green. There are 266 close orientation elements out of all 316 elements, the similarity between the two fields is 0.842.

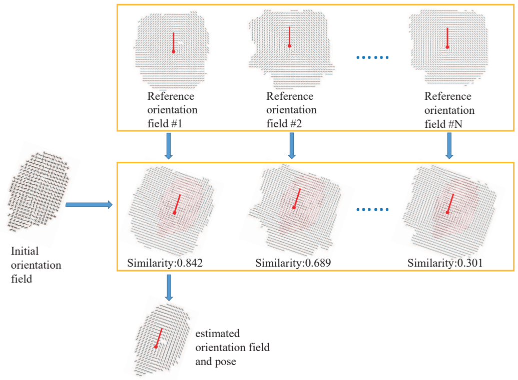


Figure 4. Example of the searching method. The reference orientation fields are transformed under different parameters and then compute the similarity between the initial one. The reference orientation field with the largest similarity is selected with its parameters as the final estimation.

mated by short time Fourier transform (STFT) [10]. Local Fourier analysis transform is applied to the query image in each 16 × 16 block and at each block we get at most two strongest orientation elements to deal with strong noises.

3.3. Exhaustive search

When the initial orientation field $\mathrm{O}_{\mathrm{ini}}$ is estimated, we propose an exhaustive searching process to find the best real orientation field in the reference database $\mathrm{D}_{\mathrm{ref}}$. Similarities between $\mathrm{O}_{\mathrm{ini}}$ and every orientation field $\mathrm{O}_{\mathrm{ref}_i}$ in $\mathrm{D}_{\mathrm{ref}}$ are computed. The computing process is stated as follows:

(1) Considering that the latent fingerprint is not always upward, the reference $\mathrm{O}_{\mathrm{ref}_i}$ is rotated by different angles. In this paper, the rotation angles range from -15 degrees to 15 degrees, every 5 degrees.

(2) The latent fingerprints usually have small areas, and $\mathrm{O}_{\mathrm{ini}}$ is far smaller than $\mathrm{O}_{\mathrm{ref}_i}$. We set a sliding window that has the same size with $\mathrm{O}_{\mathrm{ini}}$. The window slides across the rotated $\mathrm{O}_{\mathrm{ref}_i}$ and gets orientation field $\mathrm{O}_{\theta,x,i}$, the subscript θ represents for different rotation angles and x for different locations.

(3) Similarity between $\mathrm{O}_{\mathrm{ini}}$ and $\mathrm{O}_{\theta,x,i}$ is computed by the following formula:

$$\mathrm{Simi}_{\theta,x,i} = \frac{\#\text{similar orientation elements}}{\#\text{orientation elements of latent}}$$

We define two orientation elements “similar” when their difference is smaller than a threshold θ_{t}, which is experimentally set to 15 degrees. Since we retained two strongest initial orientation elements at each block, we count where either of them are similar to the reference orientation element.

(4) Compute all the similarities between the initial orientation field and reference one with different locations and rotation angles. The maximum is considered to be the final similarity between $\mathrm{O}_{\mathrm{ini}}$ and $\mathrm{O}_{\mathrm{ref}_i}$, and the corresponding rotation angle and location are recorded:

$$\mathrm{Simi}_i = \max_{\theta,x} \ \mathrm{Simi}_{\theta,x,i}$$

$$\theta_i = \arg\max_{\theta} \ \mathrm{Simi}_{\theta,x,i}$$

$$x_i = \arg\max_{x} \ \mathrm{Simi}_{\theta,x,i}$$

(5) Compute all similarities in $\mathrm{D}_{\mathrm{ref}}$ and find the closest $\mathrm{O}^*_{\mathrm{ref}}$ with the maximum similarity. And the corresponding rotation angle and location are estimated pose of the latent.

3.4. Fusion

The closest reference orientation field obtained by exhaustive search cannot be directly used as the orientation field of the latent, since they are not exactly the same. Inspired by multi-atlas algorithm in medical imaging [24], after finding the result in the atlas, a transformation is used to make the result correspond with the input image. Analogously, we try to find a way to make the reference orientation field fit the latent fingerprint. Thus, we apply a quality estimation method, and retain the orientation elements in $\mathrm{O}_{\mathrm{ini}}$ at high-quality locations. The high-quality locations are defined as where the orientation elements in $\mathrm{O}_{\mathrm{ini}}$ and $\mathrm{O}^*_{\mathrm{ref}}$ are “similar”, which means the difference is smaller than the threshold θ_{t}.

After fusion, the orientation field may change rapidly at certain locations. Therefore, Gaussian filter is applied to improve the smoothness of orientation field.

3.5. Pose estimation and boosting algorithm

In the experiment, we notice that the accuracy of the pose estimation result is not that sensitive to the scale of database, which is the main time complexity factor. Inspired by that, we further propose a boosting algorithm.

To get a smaller database $\mathrm{D}^*_{\mathrm{ref}}$, we adopt a K-mediods method for clustering as follows:

(1) All the orientation fields in $\mathrm{D}_{\mathrm{ref}}$ are aligned by rigid transform to the same coordinate, with their center at origin and the orientation at zero degree.

(2) One orientation field O_1 is randomly selected from the database $\mathrm{D}_{\mathrm{ref}}$ as the first orientation field in $\mathrm{D}^*_{\mathrm{ref}}$.

(3) For every unchecked orientation field O_j, we compute the similarity between O_j and all the orientation fields $\{\mathrm{O}_i, i = 1, 2, .., m\}$ in $\mathrm{D}^*_{\mathrm{ref}}$:

$$\mathrm{Simi}_1(\mathrm{O}_i, \mathrm{O}_j) = \frac{\sum_{l=1}^{n_{\mathrm{f}}} \cos(\mathrm{O}_i(l) - \mathrm{O}_j(l))^2}{n_{\mathrm{f}}}$$

where $\mathrm{O}_i(l)$ represents the lth orientation element and n_{f} represents the numbers of all orientation elements. If the computed similarity is larger than a threshold η, which is experimentally set to 0.75, we consider that O_j can be expressed by existing orientation fields and discard it. Otherwise, we put O_j into $\mathrm{D}^*_{\mathrm{ref}}$.

(4) Repeat (3) until all the orientation fields are checked.

After clustering, we finally get a smaller database consisting of 45 reference orientation fields. The boosting algorithm can be described as a cascade way: we first adopt exhaustive searching on the clustered database $\mathrm{D}^*_{\mathrm{ref}}$, getting the estimated pose of latent fingerprint(θ^*, x^*). While searching on the original database $\mathrm{D}_{\mathrm{ref}}$, different from exhaustive searching step (1) and (2), the rotation angles range from $\theta^* - 5$ degrees to $\theta^* + 5$ degrees, and the sliding window slides across the 60 × 60 (pixel) area with x^* as its center.

4. Experimental results

In this section, we compared the proposed method with other orientation field estimation methods on the NIST SD27 latent fingerprint database. The database consists of 258 latent fingerprints and their mated rolled fingerprints. All the 258 latent fingerprints are further classified into three subsets: 88 are identified to be of good quality, 85 of bad quality and 85 of ugly quality. The orientation field accuracy and matching accuracy are reported respectively. We also show the pose estimation accuracy and boosting effects.

4.1. Accuracy of orientation field estimation

NIST SD27 provide the manually marked orientation field and region of interest as ground truth. The accuracy is measured by the average Root Mean Square Deviation (RMSD) [25]. We compared our proposed method with: (1) CNN-based method [5] (CNN); (2) localized orientation field dictionary method [4] (LocalDict); (3) sparse coding based method [17] (Sparse Coding); (4) global orientation field dictionary method [3] (GlobalDict); (5) fingerprint orientation model based on 2D Fourier expansion [12] (FOMFE); (6) local Fourier analysis method [10] (STFT). The performances cited from the original papers are reported in Table 1. Figure 5 shows orientation fields of three latents obtained by three different methods.

Method	All	Good	Bad	Ugly
Proposed	**13.01**	10.85	13.99	**14.27**
CNN [5]	13.51	**10.76**	**13.94**	16.00
Proposed (boosting)	13.54	11.21	14.20	14.95
LocalDict [4]	14.35	11.15	15.15	16.85
Sparse Coding [17]	16.38	12.57	16.88	20.22
GlobalDict [3]	18.44	14.40	19.18	21.88
FOMFE [12]	28.12	22.83	29.09	32.63
STFT [10]	32.51	27.27	34.10	36.36

Table 1. Average root mean square deviation (RMSD) of different methods on the NIST SD27 latent database and three subsets. Methods are sorted in increasing order of RMSD on the whole database.

As shown in Table 1, the proposed method outperforms the other algorithms with an improvement of about 0.5 degrees than the state-of-the-art one. The improvement is more obvious on the subset of ugly quality. Meanwhile, we observe that the proposed method is robust to image quality as the accuracy differs least on different subsets.

To find how the scale of the reference database affects the result, we do more experiments on databases of different scales: (1) only 400 manually marked orientation fields from NIST SD4; (2) 600 orientation fields from NIST SD14 estimated by VeriFinger added to (1); (3) 9,600 orientation fields from NIST SD14 estimated by VeriFinger added to (1). The result is reported in Table 2 and an example is shown in Figure 7. As expected, the larger the database is, the more accurate the result is. The reported result in Table 1 is with the database(3), which consists of 10,000 reference fingerprints.

However, the computational complexity is linearly dependent on the database scale. We will discuss this in the following section.

4.2. Matching accuracy

To examine the contribution of orientation field estimation algorithm to the whole recognition system, we further compare different methods by conducting matching experiments. For each fingerprint pair (latent and rolled fingerprints) in NIST SD27, orientation fields are estimated seperately: the latent's orientation field is estimated by proposed

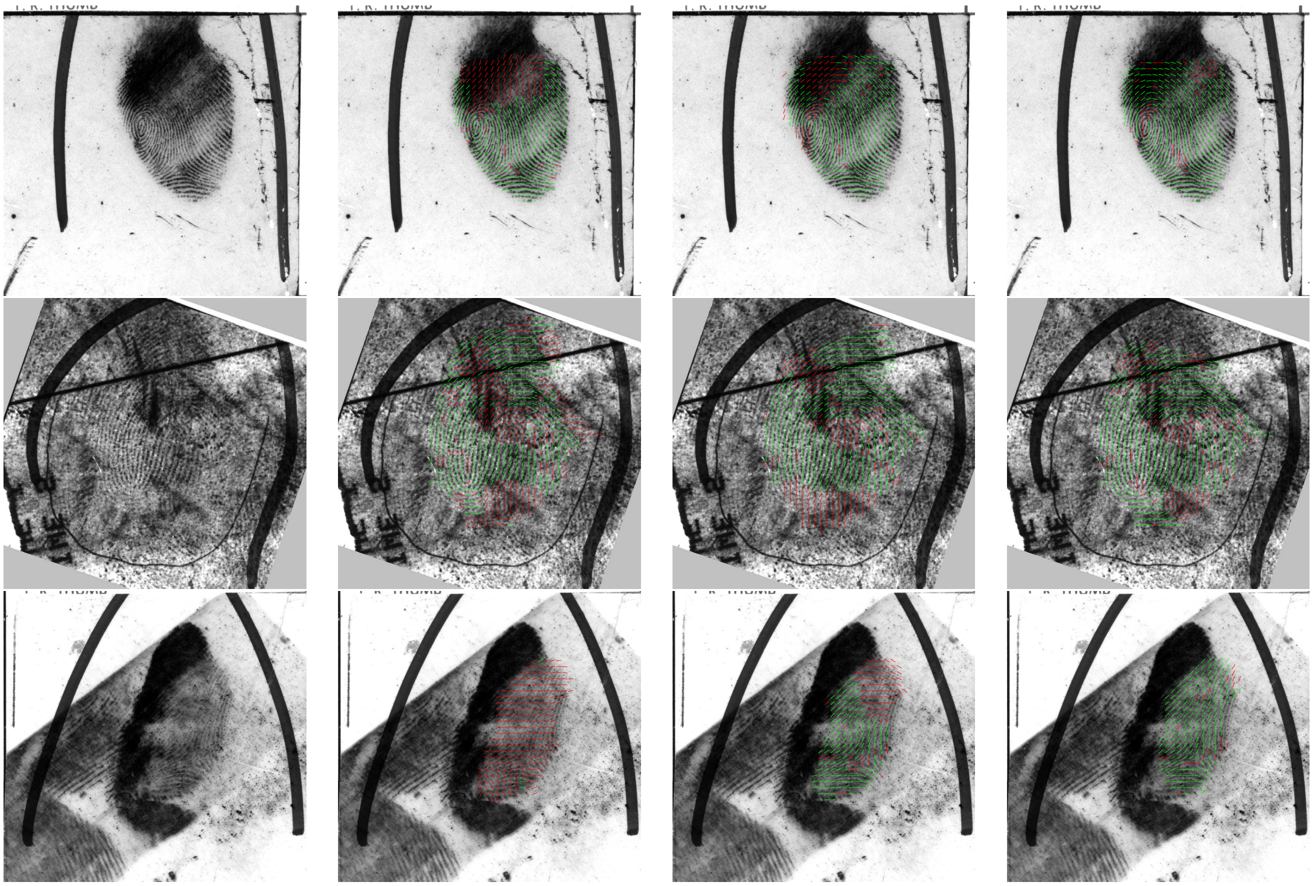

(a) (b) (c) (d)

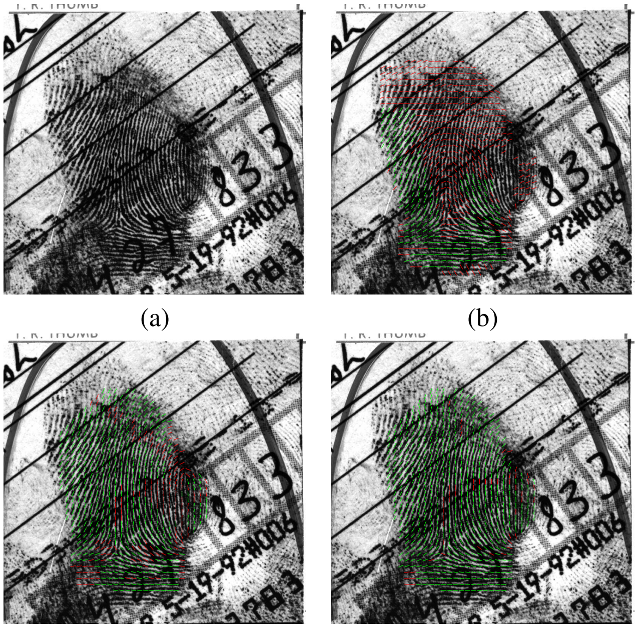
Figure 5. Comparison of orientation fields estimated by different algorithms. The latent fingerprints are shown in column (a), and in column (b)-(d) the orientation fields are estimated by global orientation field dictionary method [3], localized orientation field dictionary method [4] and proposed method, separately. The green lines are orientation field correctly estimated (difference from manual mark is below 15 degrees), while the red lines are the estimated orientation elements with an error over 15 degrees. The three latent fingerprints are from three subsets of NIST SD27.

method while the rolled fingerprint's orientation field is estimated by STFT. We adopt the Gabor filter based method in [26] to enhance the fingerprint image. The parameters of the Gabor filter are set as follows: the estimated orientation field is used as the local ridge orientation θ, the rest parameters are fixed as: $\phi = 0$, $\gamma = 1$, $\lambda = 9$ and $\sigma = 4$. The commercial software VeriFinger SDK 6.2 is used for minutiae extraction and computing the matching score between latents and rolled fingerprints. Finally, we get Cumulative Match Characteristic (CMC) curve to evaluate the matching performance. To make the matching experiment more realistic and challenging, we add 27,000 rolled fingerprints from NIST SD14 database as background. The added fingerprints are estimated and enhanced in the same way.

The CMC curves on the NIST SD27 latent fingerprint database and three subsets are shown in Figure 8. In the matching experiment, we compare the proposed method to the two dictionary-based algorithms. As we can see from these curves, the proposed method also outperforms the other ones, especially on the subset of ugly quality. The Rank-1 identification rate of the proposed method is about 1% higher than localized dictionary, and on the subset of ugly quality, the improvement is about 6%. Yet, it cannot be ignored that the reference database contains orientation fields from NIST SD14 database, which is also used as background in the matching experiment. That may increase the similarity between latents and non-mated fingerprints in NIST SD14, underestimating the matching accuracy. This can partly explain why the matching performance is slightly lower on the subset of good quality.

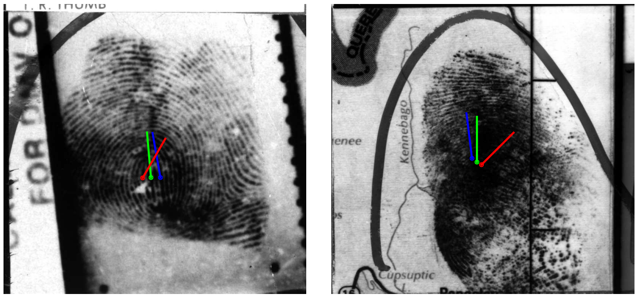
4.3. Pose estimation

As mentioned above, few methods are proposed for latent fingerprints' pose estimation, so there is no established benchmark for evaluation. Consider that the ultimate aim of pose estimation is to register latents and mated rolled fin-



(a) (b) (c) (d)

Figure 6. An example of orientation field estimation on different database scales: (a) the latent fingerprint. (b) 400 reference fingerprints. (c) 1,000 reference fingerprints. (d) 10,000 fingerprints. Note that the orientation field in (b) is completely wrong due to lack of similar reference fingerprint in the database, while (d) performs better than (c).



(a) (b)

Figure 7. Examples of different pose estimation algorithms. The manually estimated poses are marked as green lines, while the red lines are poses estimated by Hough voting based method in [4] and the blue lines are by the proposed method. The circles indicate the finger center.

gerprints to the same coordinate system, we use deviation in location and direction of mated minutiae pairs as an indicator of pose accuracy.

The evaluation method goes as follows: Each pair of mated fingerprints (from the same finger) serves as an evaluation unit. First, we use a minutiae matching algorithm to get mated minutiae pairs. Second, the poses of the fingerprints are estimated by different algorithms and then minutiae are aligned to the same coordinate system. We compute the absolute value of mean deviation in location and direction of all mated minutiae, as the performance for the algorithm. The empirical distribution functions are used for all the fingerprints in the testing dataset.

Scale	All	Good	Bad	Ugly
400	16.26	14.79	16.09	17.94
1,000	13.80	11.52	14.68	15.27
10,000	13.01	10.85	13.99	14.27

Table 2. Average root mean square deviation (RMSD) of our algorithm using different database scales on the NIST SD27 latent database and three subsets.

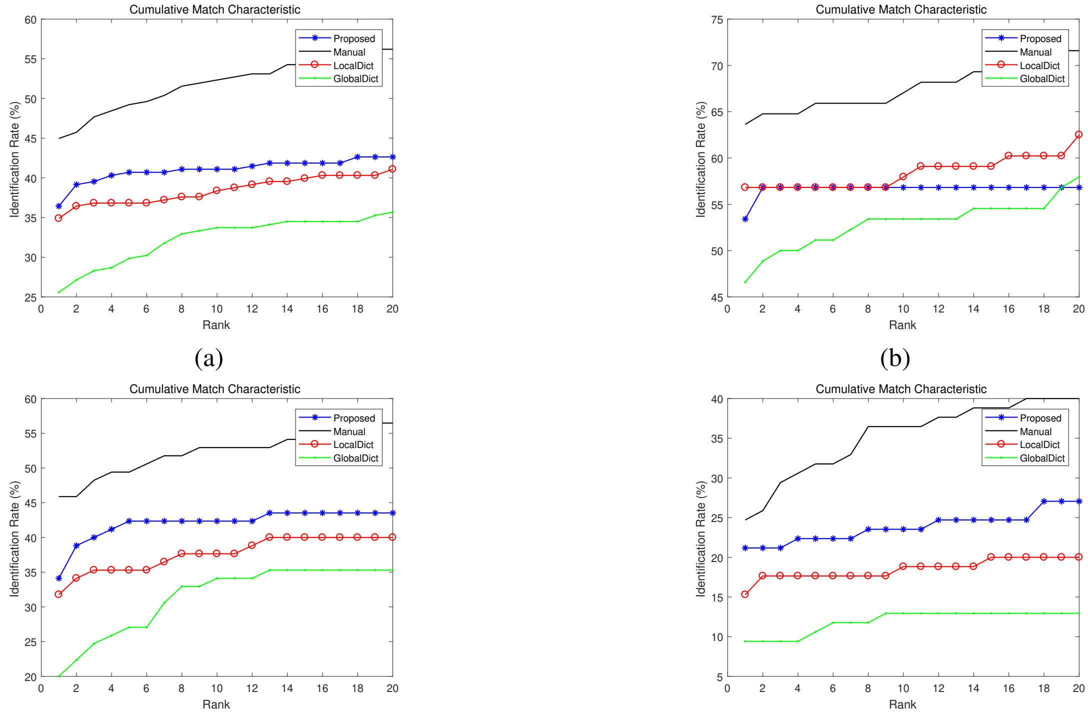
Figure 9 shows the performance of pose estimation algorithm of Yang et al. [4] and the proposed method. Over 70% mated minutiae pairs' deviation are less than 50 pixels in location. In a fingerprint image of 500 ppi resolution ratio, the average width of one ridge is about 10 pixels. That means most mated minutiae pairs' deviation are less than 5 ridges, which can extremely reduce the retrieval space. About 90% mated minutiae pairs' deviation are smaller than 15 degrees in direction. In comparison with Yang et al.'s pose estimation algorithm, our method performs slightly worse in location but much better in direction. Some examples are shoen in Figure 7.

4.4. Computational complexity

Although we do not concern the efficiency of an automatic latent fingerprint feature extraction system in reality, the computational complexity is still an important performance of one algorithm. The proposed method exhaustively searches the reference database, its computational complexity is linearly dependent on the scale of the database. Searching the closest reference orientation field for one latent fingerprint takes about 25 minutes while the database consists of 10,000 orientation fields. The runtime can be reduced to about 1 minute per latent fingerprint after boosting. All the steps are implemented in MATLAB R2017a, on a PC with 2.10 GHz CPU.

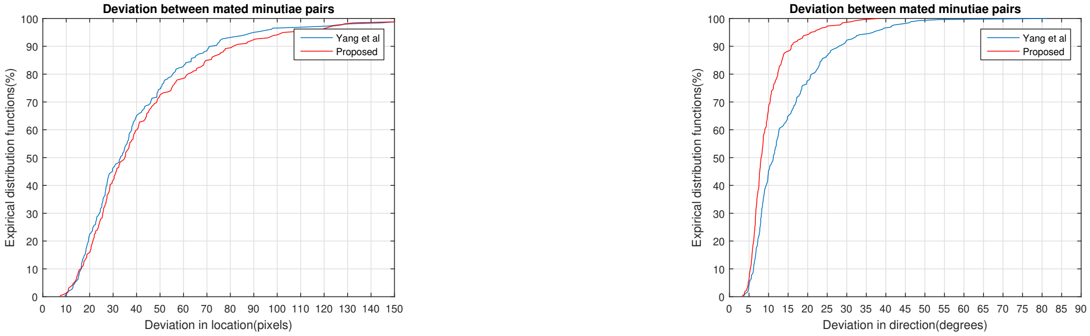
5. Summary and future work

Orientation field estimation is of great importance in fingerprint analysis. In forensic applications, an accurate estimation can extremely promote the matching performance. The state-of-the-art methods, including localized dictionary algorithm and CNN-based algorithm, have the same limitation that they split the fingerprint into small patches, and estimate the orientation field separately. Although the methods compute the correlation between adjacent patches, the integrality of the whole fingerprint is far from consideration. To deal with this problem, we proposed an exhaustive searching method, which take the fingerprint as a whole.



(a) (b) (c) (d)

Figure 8. CMC curves on NIST SD27 and three subsets. (a) All (258 latents); (b) good quality (88 latents); (c) bad quality (85 latents); (d) ugly quality (85 latents). The four curves are of different orientation field estimation approaches: manually marking, proposed method, LocalDict [4] and GlobalDict [3].



(a) (b)

Figure 9. Empirical distribution functions for deviation between mated minutiae pairs on NIST SD27 (a) in location; (b) in direction. Fingerprints are registered by two different pose estimation algorithms.

As a result, the accuracy of orientation field estimation has been improved about 0.5 degrees on NIST SD27 and the rank-1 identification rate has been improved about 1%.

The exhaustive searching method also provides a novel way for pose estimation. Experimental results show the proposed method can deal with those latent fingerprints lacking center areas, where conventional algorithms failed.

Future work may focus on: (1) better searching algorithm for even larger reference database and (2) optimizing the distribution of orientation fields in the reference database.

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